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Impact of a box with an elastic bottom on a thin liquid layer☆

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ABSTRACT

The problem of the impact of a box with an elastic bottom on a thin liquid layer is solved in a planar formulation in the shallow water approximation. Using the method of normal modes, the problem is reduced to a non-linear system of ordinary differential equations, which is solved by the Runge–Kutta method. It is shown that the elasticity of the bottom not only affects all the characteristics quantitatively but also qualitatively.

The impact of an undeformed box on a thin liquid layer has been treated using the method of matched asymptotic expansions[1] and experimental results are given in Ref. 2. The motion of a box with a constant velocity, without taking account of the retardation due to the force of the pressure of the liquid, has been considered.[1] The fall of stiff circular plates has been considered, taking account of the resistance of the medium,[3–6] under the assumption that the air or liquid flowing out from under the plate does not encounter any resistance and the pressure in the external domain is equal to zero. It has been shown[1,2] that soliton formation does not occur when a blunt body impacts on shallow water. The liquid is displaced by the body rises upwards in a strong jet, the thickness of which is comparable with the thickness of the liquid layer.

1. Basic assumptions and formulation of the problem

The problem is considered in a planar formulation. The scheme for the liquid flow and the body motion is shown in Fig. 1. In the starting position, the liquid is at rest and occupies the region $0 < y < h_0$. The bottom of the box touches the liquid surface in the range $-L < x < L$, $y = h_0$. The remaining part of the liquid surface is free. At the instant $t = 0$, the box is subjected to an impact as a result of which it starts to move downwards. The time of action of the impact is assumed to be short but finite, $0 < t < t_0$. A force $f_0(t)$ acts on the box during the impact stage and, at the end of the impact stage, the box acquires a velocity close to V_0. The box subsequently moves inertially and under the action of the force of gravity. It is assumed that the box walls are quite stiff and their deformation is not considered. The deformation of the bottom has an effect on the volume of the liquid under the box and on its flow and has a substantial effect on the form of motion of the box. The deformation of the bottom is therefore investigated in some detail. The liquid is assumed to be ideal and incompressible and its flow is assumed to be symmetrical about the Oy axis and potential. The box bottom is simulated by an elastic Euler beam. It is assumed that the thickness of the liquid layer is considerably less than the dimensions of the box: $h_0 \ll L$.

Outside the box, the pressure on the free surface of the liquid is equal to zero. Since the liquid layer is thin, we will assume that the pressure is equal to zero in the whole layer outside the box (the shallow water approximation). In order to determine the body motion, it is sufficient to know the liquid motion in the domain I (under the box).

We shall seek a solution under the assumption that the liquid displaced from under the box does not encounter the medium resistance (the pressure in the external region is equal to zero). The liquid motion in the external region then has no effect on the flow under the box and, the liquid motion in the external region is therefore not defined. The liquid flow is described by Euler's equations

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \quad \frac{\partial \upsilon}{\partial t} + u\frac{\partial \upsilon}{\partial x} + \upsilon\frac{\partial \upsilon}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g, \quad \frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (1.1)$$

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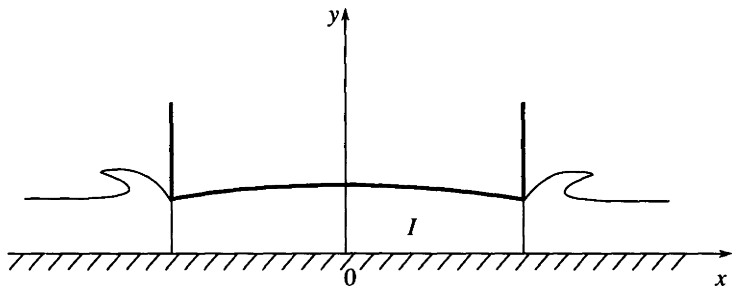

Fig. 1.

where u and v are the components of the liquid velocity, p is the pressure, ρ is the liquid density and g is the acceleration due to gravity.

The box motion is described by the relation

$$y(t) = h_0 - s(t)$$

where $s(t)$ is the depth of immersion of the box, and the position of the box bottom is defined by the expression

$$h(x, t) = h_0 - s(t) + w(x, t)$$

where $w(x, t)$ is the vertical displacement of the box bottom. From the impermeability conditions, we have

$$y = 0: v = 0; \quad y = h(x, t): h_t + uh_x = v$$

The orders of the independent variables and unknown functions in the region I are as follows:

$$x = O(L), \quad y = O(h_0), \quad t = O(h_0/V_0), \quad s(t) = O(h_0), \quad w(x, t) = O(h_0)$$

It follows from the last equation of (1.1) that $u = O(V_0L/h_0)$, and, from the first equation, we obtain that $p = O(\rho V_0^2\varepsilon^{-2})$, where $\varepsilon = h_0/L$ is a small parameter. It follows from the potentiality conditions that $\partial u/\partial y = \partial v/\partial x$. Hence, $\partial u/\partial y = O(V_0/L)$, and the term $v\partial u/\partial y$ in the first equation of (1.1) is considerably smaller than the remaining terms of the equation and can be omitted when $\varepsilon \to 0$ with a relative error $O(\varepsilon^2)$.

In the second equation of (1.1), all the terms are considerably smaller than the term $(\partial p/\partial y)/\rho$. When $\varepsilon \to 0$, we have $\partial p/\partial y = 0$ with a relative error $O(\varepsilon^2)$. Comparing the orders of the derivatives u_x and u_y, we obtain $u_y = O(\varepsilon u_x)$, and in the principal order $u_y = 0$, which corresponds to the shallow water approximation. Hence, $u = u(x, t)$, $p = p(x, t)$, and the first equation of (1.1) is written in the form

$$u_t + uu_x = -p_x/\rho$$

We set the condition

$$p(L, t) = p(-L, t) = 0$$

on the boundary of region I since, outside the region, I $p(x, t) = 0$. By virtue of the flow symmetry, $u(0, t) = 0$. According to the shallow water approximation,[7] we have

$$h_t + (uh)_x = 0$$

We will write the equation of the box motion in the form of an equation for the change in the momentum of the box (M is the mass per unit length of the box in the direction of the Oz axis)

$$\frac{d}{dt}\left(Ms'(t) - \rho_1 h_1 \int_{-L}^{L} w_t(x, t)dx\right) = Mg + f(t) - \int_{-L}^{L} p(x, t)dx$$

$$f(t) = \begin{cases} f_0(t), & t < t_0 \\ 0, & t > t_0 \end{cases} \tag{1.2}$$

The equation for the oscillations of the box bottom has the form

$$\rho_1 h_1(w_{tt} - s''(t)) + Dw_{xxxx} = p(x, t)$$

where ρ_1 and h_1 are the density of the material and the thickness of the bottom and $D = Eh_1^3/12$ is the stiffness. The conditions for the attachment of the bottom of the box are specified in the form

$$w(\pm L, t) = 0, \quad Dw''(\pm L, t) = kw'(\pm L, t)$$

where k is the stiffness of the fastening of the bottom of the box with respect to rotation. When $k=0$, we have $w''(\pm L, t) = 0$, that is, the case of a hinged fastening of the bottom. The case when $k=\infty$ corresponds to a rigid fastening of the bottom: $w'(\pm L, t) = 0$. The initial conditions are null conditions. When $t<0$, the liquid and the box are at rest.

We introduce dimensionless variables, choosing L as the length scale and, as the characteristic velocity, we choose $V_0 = t_0\bar{f}/M$, where $\bar{f}$ is the mean value of the force at the time of impact

$$\bar{f} = \frac{1}{t_0}\int_0^{t_0} f_0(t)dt$$

V_0 is the velocity which the box would acquire if there were no resistance of the liquid and gravitational acceleration. We select the quantity h_0/V_0 as the time scale. The dimensionless variables are introduced in the following manner

$$x = LX, \quad t = \frac{h_0}{V_0}T, \quad u(x,t) = \frac{V_0 L}{h_0}U(X,T), \quad p(x,t) = \frac{\rho V_0^2 L^2}{h_0^2}P(X,T)$$

$$s(t) = h_0 S(T), \quad w(x,t) = h_0 W(X,T), \quad h(x,t) = h_0 H(X,T)$$

The system of equations in dimensionless variables is written in the form

$$U_T + UU_X = -P_X$$

$$\mu(W_{TT} - S''(T)) + \beta W_{XXXX} = P(X,T)$$

$$S''(T) - 2\mu\alpha\int_0^1 W_{TT}(X,T)dX = \gamma + F(T) - 2\alpha\int_0^1 P(X,T)dX$$

$$-S'(t) + W_T(X,T) + [U(X,T)(1 - S(T) + W(X,T))]_X = 0 \tag{1.3}$$

where

$$\mu = \frac{\rho_1 h_1 h_0}{\rho L^2}, \quad \beta = \frac{Dh_1^3}{\rho V_0^2 L^6}, \quad \gamma = \frac{gh_0}{V_0^2}, \quad \alpha = \frac{\rho L^3}{Mh_0}$$

$$F(T) = \begin{cases} F_0(T), & T < T_0 \\ 0, & T > T_0 \end{cases}, \quad T_0 = \frac{t_0 V_0}{h_0}, \quad F_0(T) = \frac{h_0}{MV_0^2} f\left(\frac{h_0 T}{V_0}\right)$$

The boundary and initial conditions have the form

$$U(0,T) = 0, \quad P(1,T) = 0, \quad S(0) = S'(0) = 0, \quad W(X,0) = W_T(X,0) = 0$$

$$W(\pm 1, T) = 0, \quad \sigma W''(\pm 1, T) = W'(\pm 1, T); \quad \sigma = D/(kL)$$

2. Solution of the problem

We represent the deflection of the box bottom in the form of an expansion in eigenfunctions

$$W(X,T) = \sum_{k=0}^{\infty} A_k(T) W_k(X)$$

The eigenfunctions $W_k(X)$ are determined from the solution of the boundary-value problem

$$\frac{d^4 W_k}{dX^4} = \lambda_k^4 W_k(X); \quad W_k(\pm 1) = 0, \quad \sigma W_k''(\pm 1) = W_k'(\pm 1)$$

and have the form

$$W_k(X) = B_k\left(\cos(\lambda_k X) - \frac{\cos\lambda_k}{\operatorname{ch}\lambda_k}\operatorname{ch}(\lambda_k X)\right)$$

The eigenvalues λ_k are determined from the solution of the transcendental equation

$$2\sigma\lambda_k - \operatorname{tg}\lambda_k - \operatorname{th}\lambda_k = 0$$

When $\sigma > 1$, this equation has a single root in each of the intervals

$$[(k-1)\pi, (k-1/2)\pi], \quad k = 1, 2, \dots$$

In the case of a hinged plate

$$\lambda_k = (k-1/2)\pi, \quad W_k(X) = \cos(\lambda_k X)$$

The eigenfunctions are orthogonal and the constants B_k are chosen such that

$$\int_{-1}^{1} W_k(X) W_m(X) dX = \delta_{km}$$

where δ_{km} is the Kronecker delta.

From the last equation of system (1.3), we have

$$U(X,T) = \frac{S'(T)X - \int_0^X W_T(X',T)dX'}{H(X,T)}, \quad H(X,T) = 1 - S(T) + W(X,T) \tag{2.1}$$

Integrating the first equation of system (1.3) with respect to X, we obtain

$$\int_X^1 U_T(X',T)dX' + \frac{U^2(1,T) - U^2(X,T)}{2} = P(X,T)$$

and, substituting expression (2.1) here, we find

$$\begin{aligned}
&S''(T)\int_X^1 \frac{X'}{H(X',T)}dX' - \sum_{k=1}^{\infty} A''_k(T)\int_X^1 \frac{\Psi_k(X')}{H(X',T)}dX' + S'^2(T)\int_X^1 \frac{X'}{H^2(X',T)}dX' - \\
&- S'(T)\sum_{k=1}^{\infty} A'_k(T)\left[\int_X^1 \frac{\Psi_k(X') + X'W_k(X')}{H^2(X',T)}dX' + \frac{\Psi_k(1)}{H^2(1,T)} - \frac{X\Psi_k(X)}{H^2(X,T)}\right] + \\
&+ \sum_{k,l=1}^{\infty} A'_k(T)A'_l(T)\int_X^1 \frac{W_k(X')\Psi_l(X')}{H^2(X',T)}dX' + \frac{S'^2(T) + \Sigma^2(1,T)}{2H^2(1,T)} - \frac{S'^2(T)X^2 + \Sigma^2(X,T)}{2H^2(X,T)} = P(X,T)
\end{aligned} \tag{2.2}$$

Here,

$$\Sigma(X,T) = \sum_{k=1}^{\infty} A'_k(T)\Psi_k(X), \quad \Psi_k(X) = \int_0^X W_k(X)dX$$

We multiply Eq. (2.2) by $W_j(X)$ and integrate over the segment [0, 1]. We obtain

$$\begin{aligned}
&S''(T)I_j^{(1)}(T) + S'^2(T)\left(I_j^{(2)}(T) - \frac{I_j^{(3)}(T)}{2}\right) + \frac{[S'(T) - \Sigma(1,T)]^2}{2[1 - S(T)]^2}\Psi_j(1) - \\
&- \sum_{k=1}^{\infty} A''_k(T)I_{jk}^{(4)}(T) - S'(T)\sum_{k=1}^{\infty} A'_k(T)(I_{jk}^{(5)}(T) - I_{kj}^{(5)}(T) + I_{jk}^{(6)}(T)) + \\
&+ \sum_{k,l=1}^{\infty} A'_k(T)A'_l(T)\left(I_{jkl}^{(7)}(T) - \frac{I_{kjl}^{(7)}(T)}{2}\right) = \frac{\mu A''_j(T)}{2} + \frac{\beta\lambda_j^4 A_j(T)}{2} - \mu S''(T)\Psi_j(1)
\end{aligned} \tag{2.3}$$

Here,

$$I_j^{(1)}(T) = \int_0^1 \frac{X\Psi_j(X)}{H(X,T)}dX, \quad I_j^{(2)}(T) = \int_0^1 \frac{X\Psi_j(X)}{H^2(X,T)}dX, \quad I_j^{(3)}(T) = \int_0^1 \frac{X^2 W_j(X)}{H^2(X,T)}dX$$

$$I_{jk}^{(4)}(T) = \int_0^1 \frac{\Psi_j(X)\Psi_k(X)}{H(X,T)}dX, \quad I_{jk}^{(5)}(T) = \int_0^1 \frac{X\Psi_j(X)W_k(X)}{H^2(X,T)}dX$$

$$I_{jk}^{(6)}(T) = \int_0^1 \frac{\Psi_j(X)\Psi_k(X)}{H^2(X,T)}dX, \quad I_{jkl}^{(7)}(T) = \int_0^1 \frac{\Psi_j(X)W_k(X)\Psi_l(X)}{H^2(X,T)}dX$$

We now integrate Eq. (2.2) with respect to X, substitute the resulting expression for the force into the third equation of the system and hence we find

$$S''(T) = \Omega^{-1}\left[\Phi + \sum_{k=1}^{\infty} A_k''(T)(I_k^{(1)}(T) + \mu\Psi_k(1))\right] \tag{2.4}$$

Here,

$$\Omega = \frac{1}{2\alpha} + \int_0^1 \frac{X^2}{H(X,T)}dX$$

$$\Phi = \frac{\gamma + F(T)}{2\alpha} - \frac{S'^2(T)}{2}\int_0^1 \frac{X^2}{H^2(X,T)}dX + S'(T)\sum_{k=1}^{\infty} A_k'(T)I_k^{(3)}(T) -$$

$$-\frac{[S'(T) - \Sigma(1,T)]^2}{2[1-S(T)]^2} - \sum_{k,l=1}^{\infty} A_k'(T)A_l'(T)\left(I_{lk}^{(5)}(T) - \frac{I_{lk}^{(6)}(T)}{2}\right)$$

We substitute the expression for $S''(T)$ into Eq. (2.3) and obtain the system of equations

$$\sum_{k=1}^{\infty} B_{jk}A_k''(T) = G_j \tag{2.5}$$

Here,

$$B_{jk} = \frac{\mu\delta_{jk}}{2} - \Omega^{-1}[I_k^{(1)}(T) + \mu\Psi_k(1)][I_j^{(1)}(T) + \mu\Psi_j(1)] + I_{jk}^{(4)}(T)$$

$$G_j = \Omega^{-1}\Phi[I_j^{(1)}(T) + \mu\Psi_j(1)] - \frac{\beta\lambda_j^4 A_j(T)}{2} + S'^2(T)\left(I_j^{(2)}(T) - \frac{I_j^{(3)}(T)}{2}\right) -$$

$$-S'(T)\sum_{k=1}^{\infty} A_k'(T)\left(I_{jk}^{(5)}(T) - I_{kj}^{(5)}(T) + I_{jk}^{(6)}(T) + \frac{\Psi_j(1)\Psi_k(1)}{[1-S(T)]^2}\right) +$$

$$+\frac{S'^2(T)\Psi_j(1)}{2[1-S(T)]^2} + \sum_{k,l=1}^{\infty} A_k'(T)A_l'(T)\left(\frac{\Psi_k(1)\Psi_l(1)\Psi_j(1)}{2[1-S(T)]^2} + I_{jkl}^{(7)}(T) - \frac{I_{kjl}^{(7)}(T)}{2}\right)$$

The system of equations (2.4), (2.5) is thus obtained for the unknown functions $S(T)$ and $A_k(T)$.

We introduce the new unknown functions $V(T) = S(T)$ and $Q_k(T) = A_k'(T)$ and reduce the second order system of equations (2.4), (2.5) to the first order system of equations

$$A_k'(T) = Q_k(T); \quad Q_k'(T) = R_k, \quad R_k = \sum_{j=1}^{N} (B^{-1})_{kj}G_j$$

$$S'(T) = V(T); \quad V'(T) = \Omega^{-1}\left[\Phi + \sum_{k=1}^{N} R_k(I_k^{(1)}(T) + \mu\Psi_k(1))\right]$$

where $(B^{-1})_{kj}$ are the elements of a matrix which is the inverse of B, N is the number of modes taken into account, and, in the expressions for Φ and G_j, $S(T)$ has to be replaced by $V(T)$ and $A'(T)$ has to be replaced by $Q(T)$.

We shall solve this problem using a fourth order Runge–Kutta method.[8]

3. Motion of the undeformed box

In the case of an undeformed bottom of the box, $W=0$. It is therefore necessary to discard the second equation in system (1.3), and the second term in the third equation vanishes. The resulting system can also be solved using the Runge–Kutta method but it is also possible to obtain an analytical solution. We find

$$U(X,T) = \frac{XS'(T)}{1-S(T)}, \quad P(X,T) = \frac{1-X^2}{1-S(T)}\left(\frac{S''(T)}{2} + \frac{S'^2(T)}{1-S(T)}\right)$$

We now introduce the new independent variable $\zeta = 1 - S(T)$. The equation of motion of the box is then written in the form

$$\zeta''(1+a/\zeta) = 2a\zeta'^2/\zeta^2 - \bar{\gamma}, \quad a = 2\alpha/3, \quad \bar{\gamma} = \gamma + F(T)$$

We now make the substitutions $\zeta' = \xi(\zeta)$, $\zeta'' = \xi\xi'$, $\eta = \xi^2$ and ζ will be considered as the independent variable. We obtain an equation, solving which we find

$$\eta = C(\zeta)\zeta^4/(\zeta+a)^4$$

We find the function $C(\zeta)$ using the initial conditions.

In the impact stage, the initial conditions have the form

$$T = 0: \zeta(0) = 1, \quad \zeta'(0) = 0$$

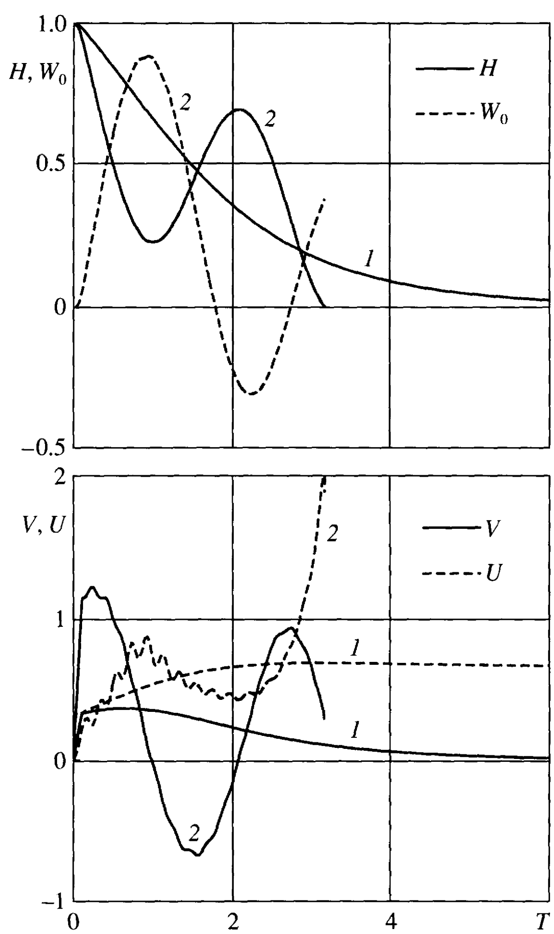

Fig. 2.

We obtain

$$C(\zeta) = 2\bar{\gamma}[1 - \zeta - 3a\ln\zeta + 3a^2(1/\zeta - 1) + a^3(1/\zeta^2 - 1)/2]$$

$$\int_{\zeta}^{1}\frac{(z+a)^2 dz}{z^2\sqrt{C(z)}} = T, \quad V(T) = -\zeta'(T) = \frac{\zeta^2\sqrt{C(\zeta)}}{(\zeta+a)^2}$$

In the free-fall phase, we find

$$C(\zeta) = V_1^2(\zeta_1 + a)^4/\zeta_1^4 + 2\bar{\gamma}[\zeta_1 - \zeta + 3a\ln(\zeta_1/\zeta) + 3a^2(1/\zeta - 1/\zeta_1) + a^3(1/\zeta^2 - 1/\zeta_1^2)/2]$$

$$\zeta_1 = \zeta(T_0), \quad \int_{\zeta}^{\zeta_1}\frac{(z+a)^2 dz}{z^2\sqrt{C(z)}} = T - T_0, \quad V_1 = V(T_0), \quad U(1,T) = \frac{\zeta\sqrt{C(\zeta)}}{(\zeta+a)^2}$$

It can be seen that the velocity of the body tends to zero as it approaches the bottom of the liquid layer and does not vanish anywhere earlier, that is, there cannot be a backward motion in the case of a rigid body. When $\zeta \to 0$, we have $V = O(\zeta)$, $T = O(\ln\zeta)$, that is, the body cannot fall after a finite time. The same paradox has been noted in earlier papers.[3] It is challenging to obtain an analytical solution in the case of a box with an elastic bottom but the calculations which have been carried out show that, as it approaches the bottom of the liquid layer, its velocity decreases and it does not succeed in reaching the right to the bottom: the box is either repelled from the bottom or the calculation becomes unstable. The calculations were stopped when the box had approached the bottom to within less than 0.1% of the depth of the liquid.

4. Numerical results

Results of numerical calculations of the dependences of the dimensionless depth of immersion h on the dimensionless time, the dimensionless deflection of the bottom at the centre W_0, the dimensionless velocity of the box V and the dimensionless velocity of the

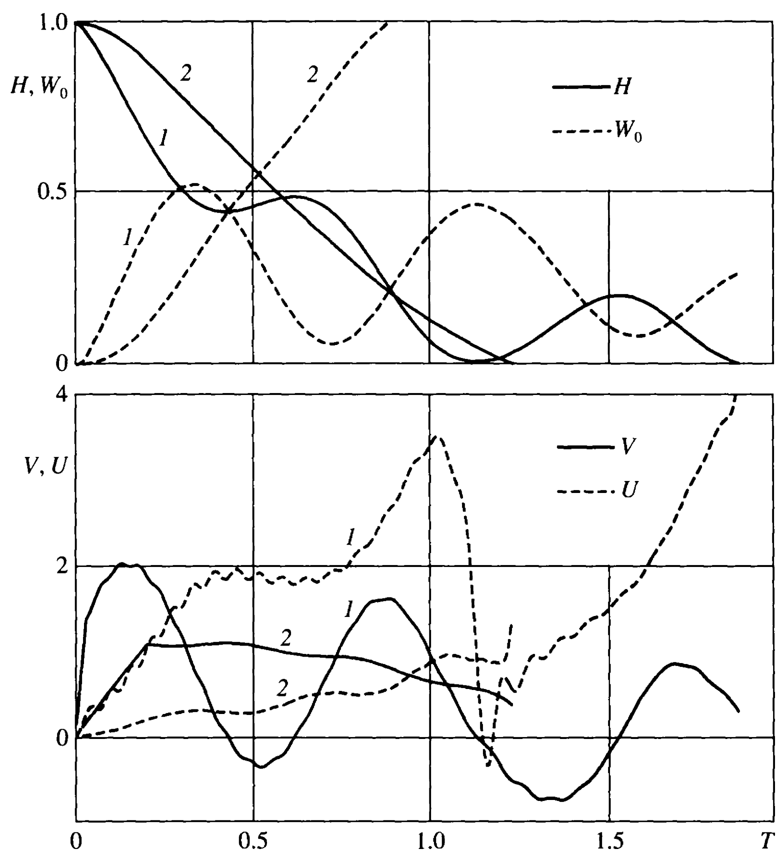

Fig. 3.

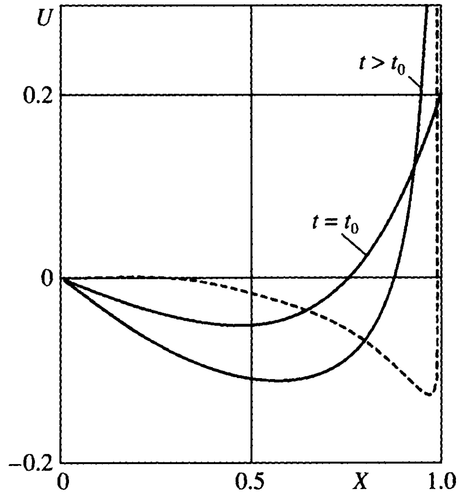

Fig. 4.

outflowing liquid are shown in Figs. 2–7 for the following parameters of the liquid, the elastic bottom of the box and other parameters

$$\rho = 10^3 \text{ kg/m}^3, \quad \rho_1 = 7.8\times10^3 \text{ kg/m}^3, \quad L = 1 \text{ m}, \quad g = 9.81 \text{ m/s}^2,$$
$$E = 2.1\times10^{11} \text{ N/m}^2, \quad t_0 = 0.01 \text{ s}, \quad h_0 = 0.1 \text{ m}$$

$h_1 = 0.02$ m (except in Fig. 7, where $h_1 = 0.03$ m) and $M = 3\times10^3$ kg (except in Fig. 6, where $M = 6\times10^3$ kg)

The force acting on the box during the impact was assumed to be constant.

In all the calculations, the first bottom deflection mode was the dominant mode. Comparison of the results for different numbers of modes showed that five modes are completely sufficient to describe the deflection of the bottom.

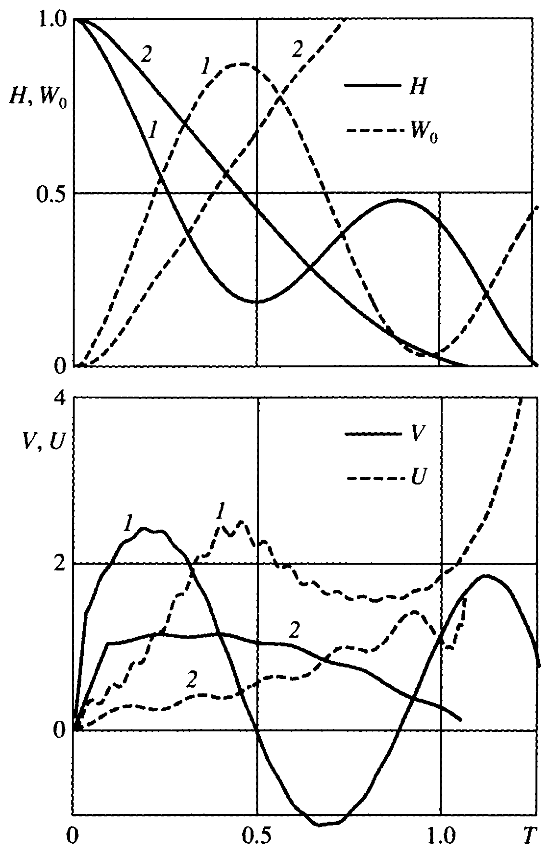

Fig. 5.

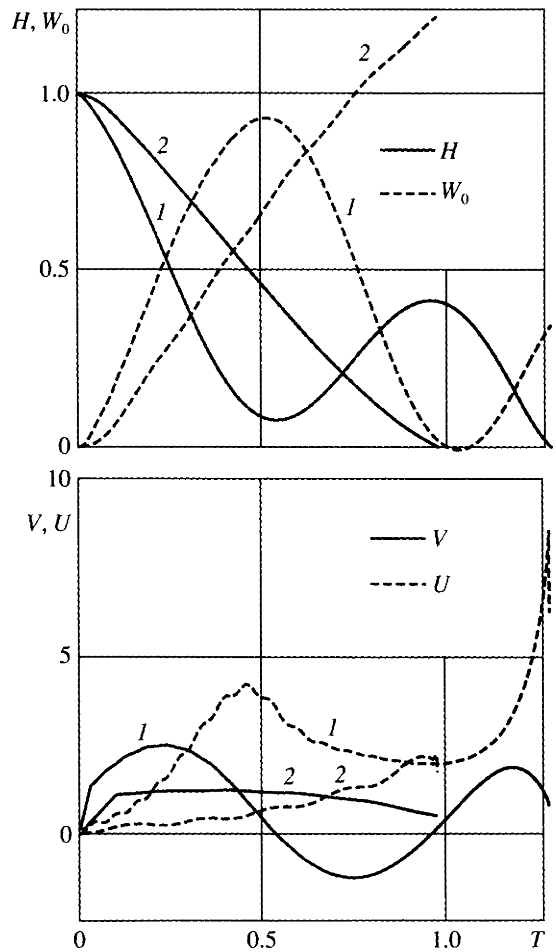

Fig. 6.

The calculations showed that the elasticity of the bottom not only considerably affects all characteristics quantitatively but, also, qualitatively. For the same force of impact, the velocity of a rigid box at the end of the impact phase is much smaller than in the case of a box with an elastic bottom. This is explained by the fact that, on account of the flexing of the bottom, the mass of liquid which flows out and its pressure are smaller, and, also, by the fact that, by virtue of the momentum conservation law, the upward deflection of the bottom thrusts the box down. As a result, the fall time of a rigid box is much longer than in the case of a box with an elastic bottom.

Unlike the case of a rigid box, the velocity of the box, the deflection of its bottom and the velocity of the outflowing liquid can change sign when the box has an elastic bottom. This can occur for a weak impact or in the case of an elastic base which is very stiff. In these cases, the motion of the box has an oscillatory form, like the motion of its bottom.

A comparison of the results of calculations for a rigid box (curves 1) and a box with an elastic bottom (curves 2) is shown in Fig. 2 for $V_0 = 1$ m/s and $\sigma = 10$. It can be seen that the box with an elastic bottom performs a backward motion and the velocity of the box and that the deflection of its bottom change sign.

In the case of a strong impact, at the end of the impact phase, the box acquires a fairly high velocity and a backward motion is not observed. It can be seen from the comparison of the cases of weak impact ($V_0 = 0.3$ m/s, curves 1) and strong impact ($V_0 = 2$ m/s, curves 2) shown in Fig. 3 for $\sigma = 10$ that, in the case of a weak impact, the box performs a backward motion twice and, in the case of its second ascent, the liquid velocity becomes negative, that is, liquid is sucked in under the box.

In the case of a strong impact, the deflection of the box bottom at its centre as it approaches the rigid surface exceeds the depth of the liquid. This does not mean that the bottom becomes detached from the liquid surface. Graphs of the velocity distribution of the liquid $U(X)$ are shown in Fig. 4 at different instants for $V_0 = 2$ m/s and $\sigma = 2$. The broken curve corresponds to the terminal stage of the fall, that is, to the position of the box near the rigid surface. It can be seen from these graphs that part of the liquid moves towards the centre in order to fill the space under the box bottom and another part flows out from under the box.

The effect of the rigidity of the fastening of the box bottom on the motion of the box and the liquid was investigated. The results of calculations for a hinged bottom and an elastic fastening when $\sigma = 10$ are qualitatively identical and are only slightly different quantitatively. When the stiffness of the fastening of the bottom is increased, the parameter σ decreases and there is a corresponding decrease in the first mode frequency. At the same time, the motion of the box becomes more stable and a backward motion is observed for a smaller impact force (see Fig. 5, where the calculation results for $V_0 = 0.3$ m/s (curves 1) and $V_0 = 1$ m/s (curves 2), $\sigma = 2$ are presented). However, in the case of a box with a rigidly fastened bottom ($\sigma = 0$), the first mode frequency is greater than in the case of an elastic fastening which leads to an enhancement of the oscillations.

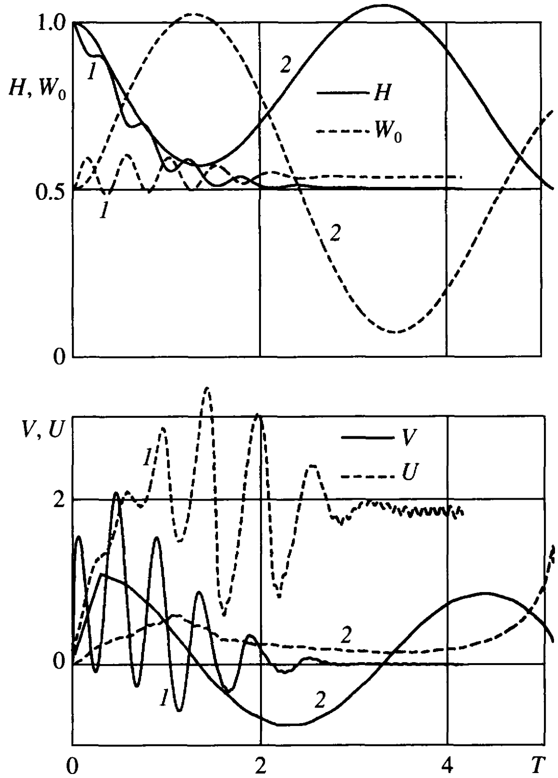

Fig. 7.

In the case of a box of large mass, a backward motion occurs for a smaller impact force. It can be seen from Eq. (1.2) that external forces act on the box: the force of gravity acts downwards and the force due to the pressure of the liquid, which is directed upwards. A greater pressure has to be applied to a body with a large mass in order to overcome the acceleration due to gravity. The calculation results for $M = 6 \times 10^3$ kg, $\sigma = 10$ and $V_0 = 0.3$ m/s (curves 1) and $V_0 = 1$ m/s (curves 2) are shown in Fig. 6. Comparison of the graphs in Figs. 2, 3, and 6 shows that an increase in the mass of the box makes its motion more stable. If the mass of the box is comparable with the mass of the bottom, the calculation becomes unstable as the box approaches the base.

An increase in the stiffness of the box bottom leads to an increase in the natural frequency of the first mode and, as a result, the oscillatory nature of the box motion is more pronounced. It can be seen from the graphs presented in Fig. 7 for $h_1 = 0.03$ m and $\sigma = 10$ that, when $V_0 = 0.3$ m/s (curves 1), the box motion has a form of high frequency oscillations, during which the oscillation amplitudes are large in the first phase and small in the second phase. When $V_0 = 3$ m/s (curves 2), the box performs a backward motion and, at the same time, the box bottom is deflected downwards and the box rises higher than the initial liquid level.

A reduction in the depth of the liquid leads to a decrease in the fall time, and the oscillatory nature of the motion is therefore less pronounced.

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